ARK 5

$$V_{OABC} = \frac{1}{6}\left|\left(\vec{AB}\times\vec{AC}\right)\cdot\vec{AO}\right|$$

$$= \frac{1}{6}\left|[20,15,12]\cdot[-3,0,0]\right|$$

$$= \frac{1}{6}\left|\left(20\cdot(-3)+15\cdot 0+12\cdot 0\right)\right|$$

$$= \frac{1}{6}\left|\left(-60\right)\right|$$

$$\underline{\underline{V_{OABC} = 10}}$$

Vi regner ut vektorproduktene for trekantene:

$$\vec{OA}\times\vec{OC} = [3,0,0]\times[0,0,5] = \begin{vmatrix} \vec{e_x} & \vec{e_y} & \vec{e_z} \\ 3 & 0 & 0 \\ 0 & 0 & 5 \end{vmatrix} = [0,-15,0]$$

$$\vec{OB}\times\vec{OC} = [0,4,0]\times[0,0,5] = \begin{vmatrix} \vec{e_x} & \vec{e_y} & \vec{e_z} \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{vmatrix} = [20,0,0]$$

$$\vec{OA}\times\vec{OB} = [3,0,0]\times[0,4,0] = \begin{vmatrix} \vec{e_x} & \vec{e_y} & \vec{e_z} \\ 3 & 0 & 0 \\ 0 & 4 & 0 \end{vmatrix} = [0,0,12]$$

Arealene:

$$F^2_{\Delta ABC} = \frac{1}{4}\left(20^2+15^2+12^2\right) = \frac{1}{4}\cdot 769$$

$$\left.\begin{aligned} F^2_{\Delta AOC} &= \frac{1}{4}\left(0^2+15^2+0^2\right) = \frac{1}{4}\cdot 225 \\ F^2_{\Delta BOC} &= \frac{1}{4}\left(20^2+0^2+0^2\right) = \frac{1}{4}\cdot 400 \\ F^2_{\Delta OAB} &= \frac{1}{4}\left(0^2+0^2+12^2\right) = \frac{1}{4}\cdot 144 \end{aligned}\right\}$$

Sum

$$\frac{1}{4}(225+400+144)$$

$$= \frac{1}{4}\cdot 769 = \underline{\underline{F^2_{\Delta ABC}}}$$

d) planet α går gjennom A, B, C

bruker normalvektor $\vec{n} = \vec{AB}\times\vec{AC} = [20,15,12]$

og et punkt i planet $A(3,0,0)$

$$20(x-3)+15(y-0)+12(z-0)=0$$

$$20x-60+15y+12z=0$$

$$\underline{\underline{\alpha:\quad 20x+15y+12z-60=0}}$$

e) plan β: $x+y-z=5$

$$\vec{n_\beta} = [1,1,-1]$$

$\vec{n_\alpha} = [20,15,12]$ Vinkelen mellom $\vec{n_\alpha}$ og $\vec{n_\beta}$:

$$\cos u = \frac{\vec{n_\alpha}\cdot\vec{n_\beta}}{|\vec{n_\alpha}|\cdot|\vec{n_\beta}|} = \frac{[20,15,12]\cdot[1,1,-1]}{\sqrt{20^2+15^2+12^2}\cdot\sqrt{1+1+1}} = \frac{20+15-12}{\sqrt{769}\cdot\sqrt{3}}$$

$$= \frac{23}{\sqrt{769}\sqrt{3}}$$

$$\underline{\underline{u = 61{,}4^\circ}}$$

f) $C(0,0,t)$ $t\neq 0$ $\vec{AC} = [-3,0,t]$ Punkt $A(3,0,0)$

Bruker $\vec{AB}\times\vec{AC} = [-3,4,0]\times[-3,0,t]$

$$= \begin{vmatrix} \vec{e_x} & \vec{e_y} & \vec{e_z} \\ -3 & 4 & 0 \\ -3 & 0 & t \end{vmatrix} = [4t,3t,12] = \vec{n}$$

Planet: $4t(x-3)+3t(y-0)+12(z-0)=0$

$$4tx-12t+3ty+12z=0$$

$$4tx+3ty+12z=12t \quad |:12t$$

$$\underline{\underline{\alpha:\quad \frac{x}{3}+\frac{y}{4}+\frac{z}{t}=1}}$$